**Global bifurcation analysis of indefinite nonlinear boundary value problems with nonlinear boundary conditions**

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This talk is devoted to the following indefinite nonlinear elliptic boundary value problem with a nonlinear boundary condition:

$$\begin{cases} -\Delta u = \lambda(m(x)u - u^2) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \lambda h(x) u^p & \text{on } \partial\Omega. \end{cases}$$

Here $\lambda \geq 0$ is a parameter, $p > 1$ is a constant, $m(x) \in C^\theta(\overline{\Omega})$ satisfies that $m(x_0) > 0$ for some $x_0 \in \Omega$ and $\int_\Omega m dx < 0$, $h \in C^{1+\theta}(\partial\Omega)$, both $m$ and $h$ may change signs, and $\mathbf{n}$ is the unit outer normal to $\partial\Omega$.

In this talk, we consider the set $(\lambda, u)$ of positive solutions. This problem possesses two trivial lines of solutions. One is $\Gamma_1 = \{(\lambda, 0) : \lambda \in \mathbb{R}\}$. The other is $\Gamma_2 = \{(0, c) : c \in \mathbb{R}\}$, where $\Gamma_2$ bifurcates from $\Gamma_1$ at $(0, 0)$. The purpose of this talk is to discuss the existence of a subcontinuum of positive solutions bifurcating from $\{(\lambda, 0) : \lambda > 0\}$ and global behavior of the subcontinuum in $\mathbb{R} \times C(\overline{\Omega})$.

The problem arises from population dynamics. Here, $u$ denotes the population density of some species inhabiting a strongly heterogenous environment, where $m(x)$ describes the local growth or decay rate of the species and $1/\lambda$ describe the diffusion rate of the species. The boundary condition is nonlinear due to the presence of $h(x)u^p$, which describes a law of the population flux on the boundary. An analytical point of view, this problem takes into account effects of absorption and explosion both in the nonlinearity.