Global bifurcation for indefinite weighted elliptic problems with nonlinear boundary conditions

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研究の目的

$\Omega \subset \mathbb{R}^N, N \geq 2$, 滑らかな有界領域

$$\begin{cases} -\Delta u = \lambda(m(x)u + g_1(x,u)) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \lambda(\sigma(x)u + g_2(x,u)) & \text{on } \partial\Omega \end{cases}$$

$\lambda \geq 0$: parameter

m, σ: sign-changing

$$\begin{cases} \displaystyle\lim_{u \to +0} \frac{g_1(x,u)}{u} = 0 \text{ uniformly in } \overline{\Omega} \\ \displaystyle\lim_{u \to +0} \frac{g_2(x,u)}{u} = 0 \text{ uniformly on } \partial\Omega \end{cases}$$

$1/\lambda$: diffusion rate

目的： 自明枝 $(\lambda, 0)$ からの正値分岐解の存在とその大域的な挙動の考察

線形化固有値問題

$$\begin{cases} -\Delta\varphi = \lambda m(x)\varphi & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \lambda\sigma(x)\varphi & \text{on } \partial\Omega \end{cases}$$

$\lambda \geq 0$ における principal eigenvalue の存在について,

$$\int_\Omega m\,dx + \int_{\partial\Omega} \sigma\,ds$$

$$\begin{cases} \geq 0 & \Longrightarrow \ \lambda = 0 \text{ のみ} \\ < 0 & \Longrightarrow \ \lambda = 0,\ \lambda = \lambda_1(m,\sigma) > 0 \end{cases}$$

[Brown-Lin '80, U.'06]

仮定： $\int_\Omega m\,dx + \int_{\partial\Omega} \sigma\,ds < 0$

主結果

$\mathcal{S} := \{(\lambda, u) \in [0, \infty) \times C(\overline{\Omega}) :$

u is a positive solution for some $\lambda\}$

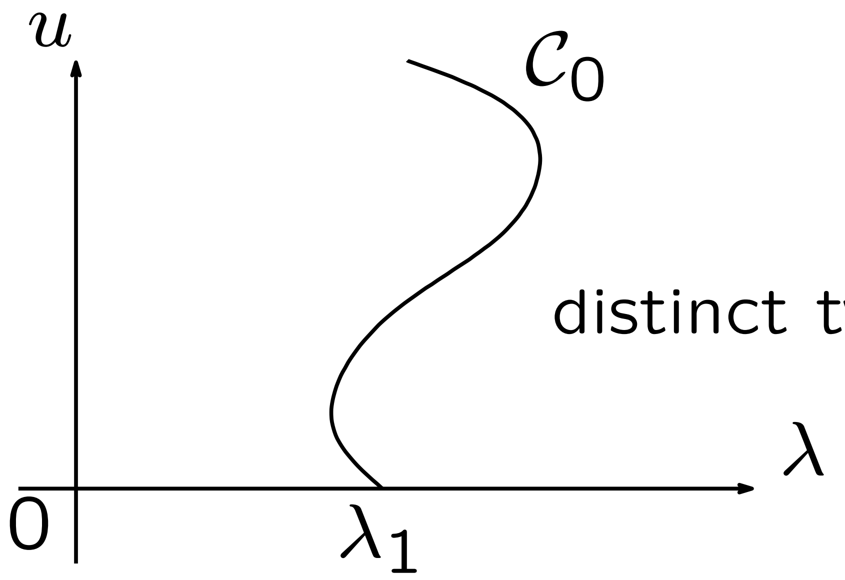
定理 $\overline{\mathcal{S}}$ は $(\lambda_1, 0)$ から分岐するsubcontinuum $\mathcal{C}_0$ を含む．分岐点 $(\lambda_1, 0)$ は $\lambda > 0$ においてただ一つ.

さらに，$\mathcal{C}_0$ は次の二者択一的である：

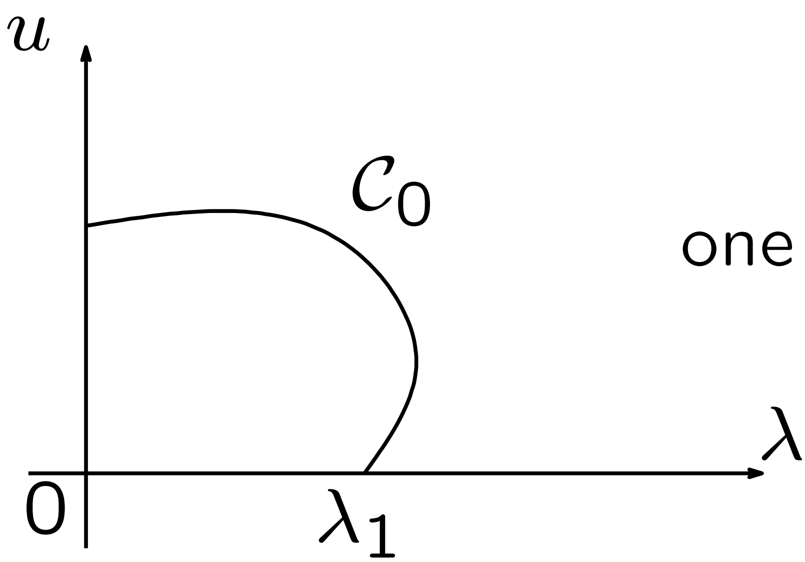
(i) unbounded in $(0, \infty) \times C(\overline{\Omega})$

(ii) connects $(0, c)$ for some constant $c > 0$

結果の概念図

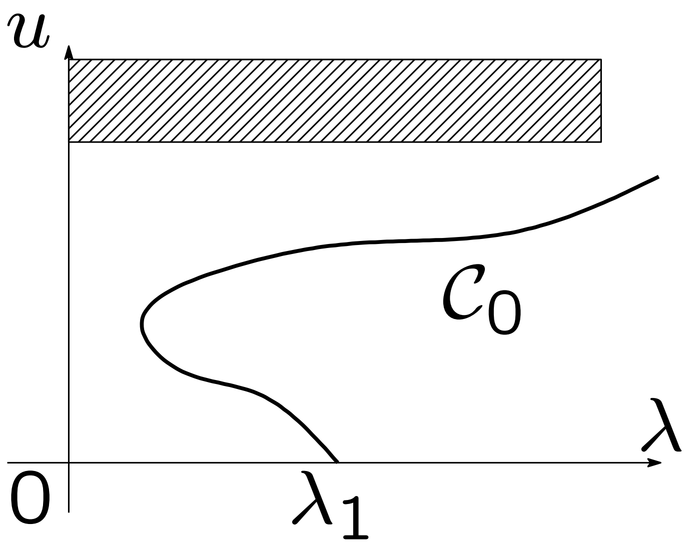


distinct two components



one component

多重性



$$\begin{cases} -\Delta u = \lambda(m(x)u - u^2) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \lambda h(x) u^p & \text{on } \partial\Omega. \end{cases}$$

$$1 < p < 2, \quad |\Omega| > m_p \int_{\partial\Omega} h ds,$$

$$2 \leq N \leq 5, \quad 1 < p < p_0(N),$$

$$p_0(N) = 1 + \frac{4}{N + 3 + \sqrt{N^2 - 2N + 25}},$$

$$\int_{\partial\Omega} h \, \varphi_1^{p+1} ds > 0$$

証明の概略

[Rabinowitz '71] $u = \lambda \mathcal{K} u + \mathcal{B}(\lambda, u)$

我々の operator equation:

$$u = \mathcal{K}_\Omega((\lambda m + M_1)u) + \mathcal{K}_{\partial\Omega}((\lambda\sigma + M_2)u) + \mathcal{B}(\lambda, u)$$

$M_1, M_2 > 0$, constant,

$$v = K_\Omega w \iff \begin{cases} (-\Delta + M_1)v = w & \text{in } \Omega \\ (\partial_{\mathbf{n}} + M_2)v = 0 & \text{on } \partial\Omega \end{cases}$$

$$v = K_{\partial\Omega}\psi \iff \begin{cases} (-\Delta + M_1)v = 0 & \text{in } \Omega \\ (\partial_{\mathbf{n}} + M_2)v = \psi & \text{on } \partial\Omega \end{cases}$$

Unilateral global bifurcation theory

$$\mathcal{L}(\lambda)u = \mathcal{B}(\lambda, u)$$

$$\Sigma := \{\lambda : \dim N[\mathcal{L}(\lambda)] \geq 1\} : \quad \text{discrete}$$

我々の場合,

$$\mathcal{L}(\lambda)u :=$$

$$u - [\mathcal{K}_\Omega((\lambda m + M_1)u) + \mathcal{K}_{\partial\Omega}((\lambda\sigma + M_2)u)]$$

定理 [López-Gómez '00] Assume

$$\lambda_1 \in \Sigma, \qquad \dim N[\mathcal{L}(\lambda_1)] = 1,$$

$$\mathcal{L}'(\lambda_1)(N[\mathcal{L}(\lambda_1)]) \oplus R[\mathcal{L}(\lambda_1)] = C(\overline{\Omega})$$

Then, each of components $\mathcal{C}_+, \mathcal{C}_-$ bifurcating from $(\lambda_1, 0)$ satisfies the alternative conditions:

(**i**) unbounded, (**ii**) $\exists \lambda^* \in \Sigma \setminus \{\lambda_1\}$ s.t. $(\lambda^*, 0) \in \mathcal{C}_+$ $(\mathcal{C}_-)$, or

(**iii**) contains a point $(\lambda, u) \in \mathbb{R} \times (Y \setminus \{0\})$, where Y is the complement of $N[\mathcal{L}(\lambda_1)]$.