convex-concave 混合型境界値問題の解構造における不定符号係数の役割について

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Problem

$$\begin{cases} -\Delta u = \lambda(m(x)u + a(x)|u|^{p-2}u) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathrm{n}} = \lambda b(x)|u|^{q-2}u & \text{on } \partial\Omega. \end{cases}$$

- $\Omega \subset \mathbb{R}^N$, a bounded smooth domain, $N \geq 2$.
- $\lambda \in \mathbb{R}$ is a parameter.
- m, a, b may change sign.
- $1 < q < 2 < p$.
- $p < \frac{2N}{N-2}$ if $N > 2$.

convex-concave nonlinearities with indefinite weights:

$$\begin{cases} \mathcal{L}_\lambda u \stackrel{\text{def}}{=} (-\Delta - \lambda m)u = \lambda a|u|^{p-2}u & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathrm{n}} = \lambda b|u|^{q-2}u & \text{on } \partial\Omega, \end{cases}$$

where $1 < q < 2 < p$, and $p < \frac{2N}{N-2}$ if $N > 2$.

- Ambrosetti-Brezis-Cerami (1994)
  convex-concave nonlinearities in Ω with constant coefficients under $u = 0$ on $\partial\Omega$

- Brown-Wu (2007)
  convex-concave nonlinearities in Ω with indefinite coefficients under $u = 0$ on $\partial\Omega$

- Wu (2008)
  convex nonlinearity on $\partial\Omega$ and concave nonlinearity in Ω with indefinite coefficients

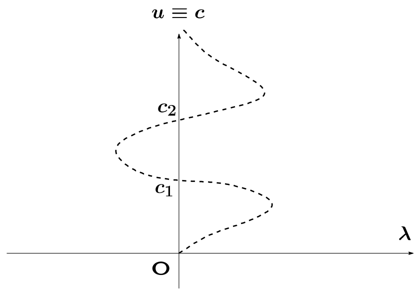
$$\begin{cases} (-\Delta + 1)u = \lambda a|u|^{q-2}u & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \mathrm{n}} = b|u|^{p-2}u & \text{on } \partial\Omega, \end{cases}$$

$1 < q < 2 < p < \frac{2(N-1)}{N-2}$.

Purpose: to consider existence, nonexistence and multiplicity of non-trivial nonnegative solutions and their asymptotic profiles when $\lambda \to 0^+$ by bifurcation and variational techniques (cf. Wu (2008)).

Case: a, b change sign, and

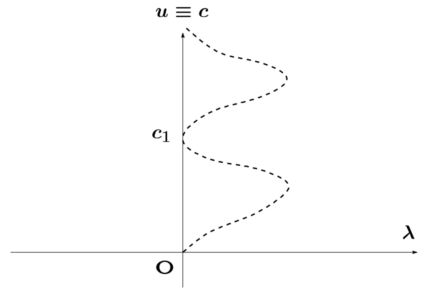
$$\int_\Omega m > 0 > \int_\Omega a, \quad 0 > \int_{\partial\Omega} b > -K(m, a)$$



bifurcation
variational

Case: a, b change sign, and

$$\int_\Omega m > 0 > \int_{\partial\Omega} b = -K(m, -m)$$



bifurcation
variational

Nehari manifolds and fibering map methods (variational approach)

$$I_\lambda(u) \overset{\text{def}}{=} \frac{1}{2}E_\lambda(u) - \frac{\lambda}{p}A(u) - \frac{\lambda}{q}B(u), \quad u \in H^1(\Omega),$$

$$E_\lambda(u) = \int_\Omega (|\nabla u|^2 - \lambda m u^2),$$
$$A(u) = \int_\Omega a|u|^p, \quad B(u) = \int_{\partial\Omega} b|u|^q.$$

Coercive properties:

Prop.

(1) Assume $\int_{\partial\Omega} b < 0$. Given $\lambda^* > 0$ small, then
$E_\lambda(u) \geq C_0\|u\|^2, \quad \forall\lambda \in (0, \lambda^*)$ and $\forall u \in B_0^+$.

(2) Assume $\int_\Omega a < 0$. Given $\lambda_* > 0$ small, then
$E_\lambda(u) \geq D_0\|u\|^2, \quad \forall\lambda \in (0, \lambda_*)$ and $\forall u \in A_0^+$.

Nehari manifold:

$$\mathcal{N}_\lambda \overset{\text{def}}{=} \{u \neq 0;\ \langle I'_\lambda(u), u\rangle = 0\ \}$$
$$= \{u \neq 0;\ E_\lambda(u) = \lambda(A(u) + B(u))\ \}$$

Fibering map: For $u \neq 0$, define, $t > 0$,

$$j_u(t) \overset{\text{def}}{=} I_\lambda(tu) = \frac{t^2}{2}E_\lambda(u) - \frac{\lambda t^p}{p}A(u) - \frac{\lambda t^q}{q}B(u).$$

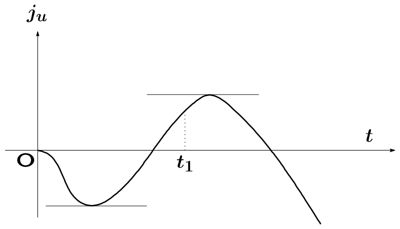
Prop. $j'_u(t) = 0 \iff tu \in \mathcal{N}_\lambda$.

Put $\mathcal{N}_\lambda^\pm = \{u \in \mathcal{N}_\lambda : \langle J'_\lambda(u), u\rangle \gtrless 0\}$, where $J_\lambda(u) = \langle I'_\lambda(u), u\rangle$. Assume $j'_u(t) = 0$. Then,

$$j''_u(t) \gtrless 0 \iff tu \in \mathcal{N}_\lambda^\pm.$$

Prop. For any critical point of I_λ in $\mathcal{N}_\lambda^\pm$ then it is a critical point of I_λ.

Pick $u \in {E_\lambda}^+ \cap A^+ \cap B^+$, and then, observe



if $j_u(t_1) > 0$ for some $t_1 > 0$.

<u>Prop.</u> If $0 < \lambda < \lambda_s$ then $j_u(t) > 0$ somewhere for ANY $u \in {E_\lambda}^+ \cap A^+ \cap B^+$.

To show:

$$\inf_{\mathcal{N}_\lambda^+ \cap B^+} I_\lambda \qquad \text{and} \qquad \inf_{\mathcal{N}_\lambda^- \cap A^+} I_\lambda$$

are achieved under the conditions

$$\int_{\partial\Omega} b < 0 \qquad \text{and} \qquad \int_\Omega a < 0,$$

respectively.