Loop components of nontrivial nonnegative solutions for indefinite concave-convex problems

梅津健一郎（茨城大教育）
Uriel Kaufmann (Univ. Nacional de Córdoba)
Humberto Ramos Quoirin (Univ. de Santiago de Chile)

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問題

Ω は滑らかな境界をもつ有界領域.

$$(P_\lambda) \qquad \begin{cases} -\Delta u = \lambda a(x) u^q + b(x) u^p & \text{in} \quad \Omega \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

- $\lambda \in \mathbb{R}$, parameter
- $a, b \in C(\overline{\Omega})$ は符号変化する
- $0 < q < 1 < p < \frac{N+2}{N-2}$ (subcritical and concave-conve)
- ν は $\partial\Omega$ 上外向き単位法線ベクトル場

u は (P_λ) の非自明解 $\iff$ $u \geq 0$ and $u \not\equiv 0$

ABC の結果

Ambrosetti-Brezis-Cerami (1994)

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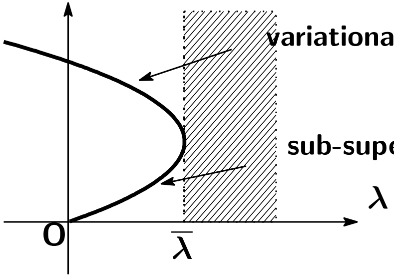
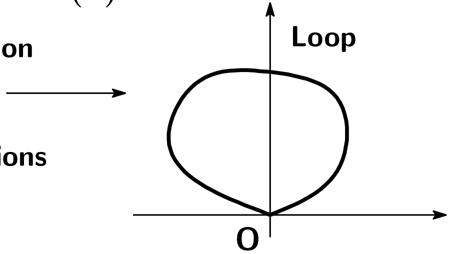


indefinite
$\lambda a(x) u^q$



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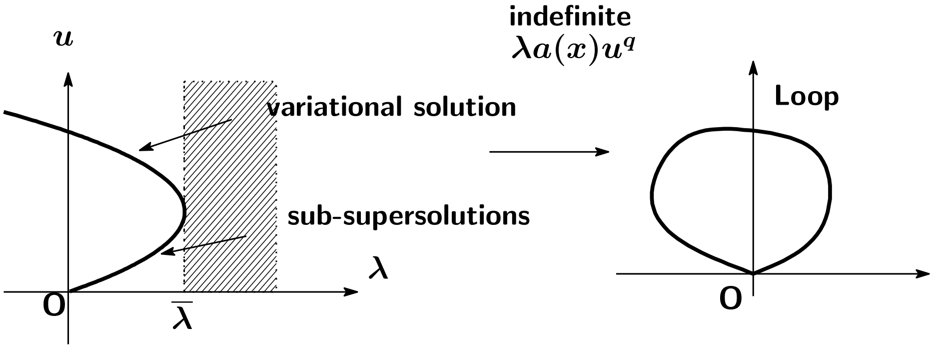

$$\frac{\partial u}{\partial \nu} = 0 \implies \int_{\partial\Omega} (\lambda u^q + u^p) dx = 0 \ (\lambda \geq 0), \text{ impossible}$$

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'ε-regularization' ($\varepsilon > 0$):

$$(P_{\lambda,\varepsilon}) \quad \begin{cases} -\Delta u = \lambda a(x)(u+\varepsilon)^{q-1}u + b(x)u^p & \text{in} \quad \Omega \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega \end{cases}$$

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$u = 0$ での線形化固有値問題 $\begin{cases} -\Delta \phi = \lambda a(x)\varepsilon^{q-1}\phi & \text{in } \Omega \\ \frac{\partial \phi}{\partial \nu} = 0 & \text{on } \partial\Omega \end{cases}$

正の主固有値 $\boxed{\lambda = \lambda_\varepsilon}$ と，自明な主固有値 $\boxed{\lambda = 0}$ ，さらに
$\lim_{\varepsilon \to +0} \lambda_\varepsilon = 0$

つづき

$\boldsymbol{\lambda = 0}$**:** $\begin{cases} -\Delta u = b(x)u^p & \textbf{in } \Omega \\ \frac{\partial \phi}{\partial \nu} = 0 & \textbf{on } \partial\Omega \end{cases}$

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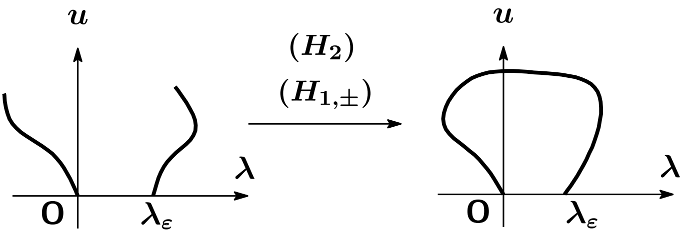

(H_2): **Amann, López-Gómez (1998)**

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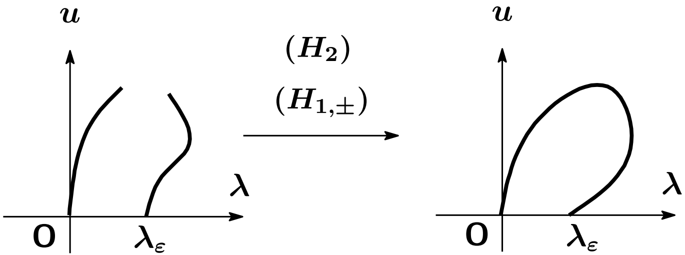
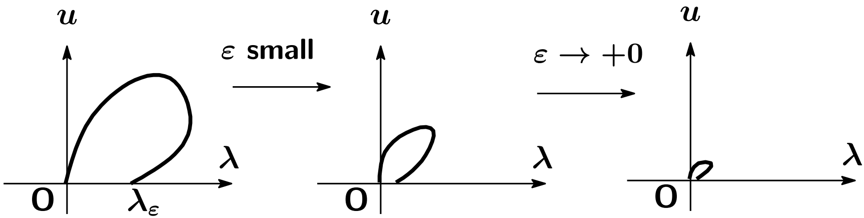


$\int_\Omega bdx \geq 0$ の場合 -scaling and regularization-



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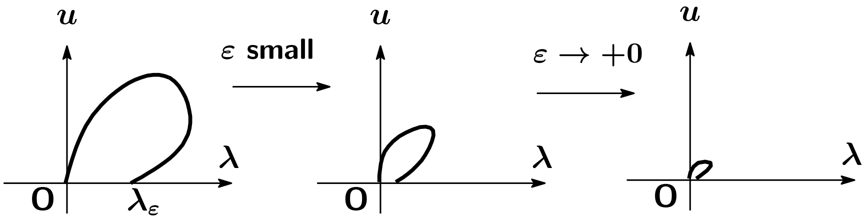

'scaling' $\boxed{v = \lambda^{-\frac{1}{p-q}} u}$, $\mu = \lambda^{\frac{p-1}{p-q}}$

(Q_μ)

$$\begin{cases} -\Delta v = \mu\left(a(x)v^q + b(x)v^p\right) & \text{in} \quad \Omega \\ \frac{\partial v}{\partial \nu} = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

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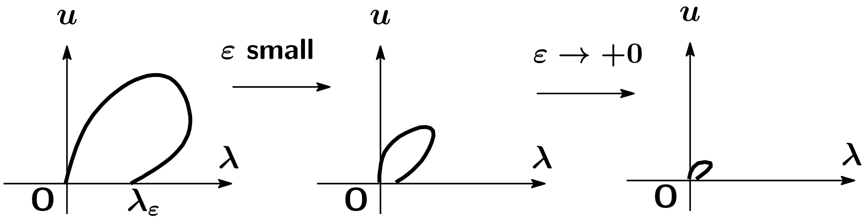

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'regularization'

$$(Q_{\mu,\varepsilon}) \qquad \begin{cases} -\Delta v = \mu \left(a(x)(v+\varepsilon)^{q-1} v + b(x) v^p \right) & \text{in} \quad \Omega \\ \frac{\partial v}{\partial \nu} = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

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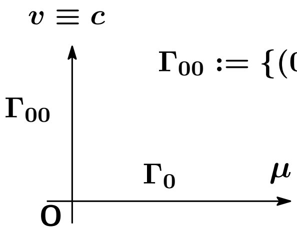

$$\Gamma_{00} := \{(0, c) : c > 0 \text{ const.}\}$$

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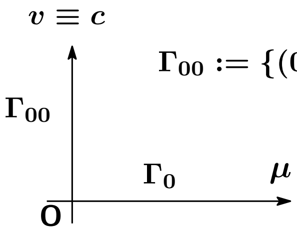

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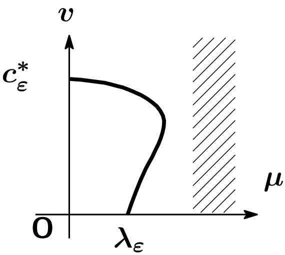
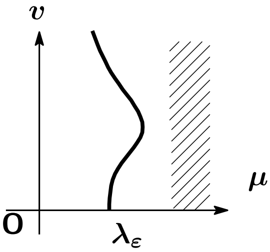


$(Q_{\mu,\varepsilon})$

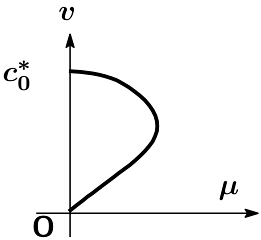


$$\int_\Omega \{a(x)(c+\varepsilon)^{q-1}c + b(x)c^p\}dx = 0 \implies c = c_\varepsilon^*$$

もうひとつの Loop

$\varepsilon \to +0$,

bifurcation from ∞ at $\mu = 0$



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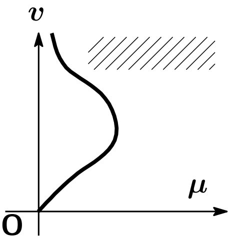

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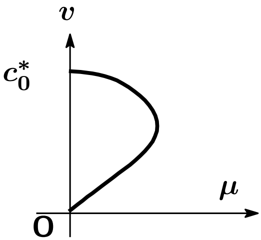

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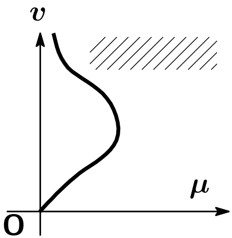
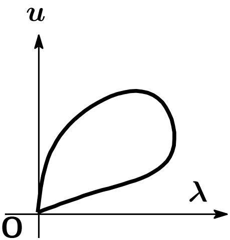


$$u = \lambda^{\frac{1}{p-q}} v$$



ありがとうございました.