[1] 次の変数分離形の一般解を求めよ.

(1) $\dfrac{dy}{dx} = -2y, \quad y = Ce^{-2x}$

(2) $\dfrac{dy}{dx} = e^{x-y}, \quad y = \log(e^x + C)$

(3) $\dfrac{dy}{dx} = x\sqrt{y}, \quad y = (\frac{x^2}{4} + C)^2$

(4) $\dfrac{dy}{dx} = (1-2x)y^2, \quad y = \frac{1}{x^2-x+C}$

(5) $\dfrac{dy}{dx} = y^2 - 3y + 2, \quad y = \frac{2-Ce^x}{1-Ce^x}$

(6) $\dfrac{dy}{dx} = \left(\dfrac{1+y}{1+x}\right)^2, \quad y = \frac{x-C(1+x)}{1+C(1+x)}$

(7) $\dfrac{dy}{dx} = \dfrac{xy}{x^2-1}, \quad y^2 = C(x^2-1)$

(8) $\dfrac{dy}{dx} = \dfrac{2+y^2}{1+x^2}, \quad y = \sqrt{2}\tan(\sqrt{2}\tan^{-1}x + C)$

(9) $\dfrac{dy}{dx} = e^{-y}\log(1-3x), \quad y = \log(-x + (x - \frac{1}{3})\log(1-3x) + C)$

(10) $\dfrac{dy}{dx} = y^2 \sin 2x, \quad y = \frac{2}{\cos 2x + C}$

(11) $\dfrac{dy}{dx} = x(y+y^2), \quad y = \frac{Ce^{\frac{x^2}{2}}}{1-Ce^{\frac{x^2}{2}}}$

[2] 次の同次形の一般解を求めよ.

(1) $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}, \quad \tan^{-1}(\frac{y}{x}) = \log\sqrt{x^2+y^2} + C$

(2) $\dfrac{dy}{dx} = \dfrac{2xy}{x^2-y^2}, \quad (\frac{y}{x})^2 = C(1+(\frac{y}{x})^2)^3 x^2$

(3) $(xy - x^2)\dfrac{dy}{dx} = y^2, \quad \frac{y}{x} = \log|y| + C$

[3] 指示された変換により変数分離形に帰着して一般解を求めよ.

(1) $\dfrac{dy}{dx} = x + y \quad (u = x+y), \quad y = -x + Ce^x - 1$

(2) $\dfrac{dy}{dx} = 2x(x^2+y) \quad (u = x^2+y), \quad y = -1 - x^2 + Ce^{x^2}$

(3) $(x^2y + x)\dfrac{dy}{dx} + xy^2 - y = 0 \quad (u = xy), \quad \frac{1}{xy} = \log|\frac{y}{x}| + C$

(4) $\dfrac{dy}{dx} = \dfrac{1}{\cos(x+y)} \quad (u = x+y), \quad y = \tan\frac{x+y}{2} + C$