6/21 解析入門.

問 1. $p = \frac{m_1}{n_1} > 0$, $q = \frac{m_2}{n_2} > 0 \Rightarrow a^p \cdot a^q = a^{p+q}$ を示せ.

$$a^p \cdot a^q = a^{\frac{m_1 n_2}{n_1 n_2}} \cdot a^{\frac{n_1 m_2}{n_1 n_2}}$$

$$= \left(a^{\frac{1}{n_1 n_2}}\right)^{m_1 n_2} \cdot \left(a^{\frac{1}{n_1 n_2}}\right)^{n_1 m_2}$$

$$= \left(a^{\frac{1}{n_1 n_2}}\right)^{m_1 n_2 + n_1 m_2} = a^{\frac{m_1 n_2 + n_1 m_2}{n_1 n_2}} = a^{\frac{m_1}{n_1} + \frac{m_2}{n_2}}$$

問 2: $(3^2 \cdot 5^5)^{\frac{2}{3}} \times 5^{-\frac{4}{3}} \div \sqrt[3]{3} = 75$.

問 1 (1) $\log_3 81 = 4$

(2) $\log_a 1 = 0$

(3) $\log_a a = 1$

(4) $\log_{\frac{1}{4}} 16 = -2$.

問 2 $\log_{10} 120$ を $\log_{10} 2 = \alpha$, $\log_{10} 3 = \beta$ で表せ.

$$\log_{10} 120 = \log_{10} 12 + \underbrace{\log_{10} 10}_{1}$$

$$= \log_{10} 3 + \underbrace{\log_{10} 4}_{2 \log_{10} 2} + 1 = \beta + 2\alpha + 1$$