

4/12 '06 小 答 解

答 解

(1) $\int x \log x \, dx$

(2) $\int x^2 e^{-x} \, dx$

(3) $\int \tan^{-1} x \, dx$

解答

$$\begin{aligned} 1. \quad \int x \log x \, dx &= \frac{x^2}{2} \log x - \int \frac{x}{2} \, dx \\ &= \frac{x^2}{2} \log x - \frac{x^2}{4} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int x^2 e^{-x} \, dx &= -x^2 e^{-x} + \int 2x e^{-x} \, dx \\ &= -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} \, dx \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C \end{aligned}$$

$$3. \quad \int \tan^{-1} x \, dx$$

$$= \int (x)' \tan^{-1} x \, dx$$

$$= x \tan^{-1} x - \int \frac{x}{x^2+1} \, dx$$

$$t = x^2+1 \quad x < t < x \quad \frac{dt}{dx} = 2x \quad \therefore \frac{dx}{dt} = \frac{1}{2x}$$

$$= x \tan^{-1} x - \int \frac{x}{t} \cdot \frac{1}{2x} \, dt$$

$$= x \tan^{-1} x - \int \frac{1}{2t} \, dt = x \tan^{-1} x - \frac{1}{2} \log |t| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \log |x^2+1| + C$$