1. 次の定積分を求めよ

(a) $\displaystyle\int_1^2 \frac{x}{x^2-2x+2}\,dx$

(b) $\displaystyle\int_1^e \sqrt{x}\log x\,dx$

2. 定積分の性質を用いることによって次の不等式を示せ.

$$\int_0^1 5x^4 e^{x-1}\cos x\,dx \leqq 1$$

K.U.

解答

1. (a) $\displaystyle\int_1^2 \frac{x}{x^2-2x+2}\,dx$

$$= \int_1^2 \left(\frac{\frac{1}{2}(2x-2)}{x^2-2x+2} + \frac{1}{x^2-2x+2} \right) dx$$

$$= \left[\frac{1}{2}\log|x^2-2x+2| + \tan^{-1}(x-1) \right]_1^2$$

$$= \frac{1}{2}\log 2 + \underbrace{\tan^{-1} 1}_{\frac{\pi}{4}} - \frac{1}{2}\underbrace{\log 1}_{0} - \underbrace{\tan^{-1} 0}_{\overset{\shortparallel}{0}} = \frac{1}{2}\log 2 + \frac{\pi}{4}$$

(b) $\displaystyle\int_1^e \sqrt{x}\log x\,dx$

$$= \left[\frac{2}{3}x^{\frac{3}{2}}\log x \right]_1^e - \frac{2}{3}\int_1^e \overbrace{x^{\frac{3}{2}}\cdot\frac{1}{x}}^{=x^{\frac{1}{2}}}\,dx$$

$$= \frac{2}{3}e^{\frac{3}{2}} - 0 - \frac{2}{3}\left[\frac{2}{3}x^{\frac{3}{2}} \right]_1^e$$

$$= \frac{2}{3}e^{\frac{3}{2}} - \frac{4}{9}e^{\frac{3}{2}} + \frac{4}{9} = \frac{2}{9}e^{\frac{3}{2}} + \frac{4}{9}$$

2. $\displaystyle\int_0^1 5x^4 e^{x-1}\cos x\,dx$

$$\leq \int_0^1 |5x^4 e^{x-1}\cos x|\,dx$$

$$\leq \int_0^1 5x^4 e^{x-1}\,dx \qquad 0 \leq e^{x-1} \leq 1,\ (0 \leq x \leq 1).$$

$$\leq \int_0^1 5x^4\,dx = \left[x^5 \right]_0^1 = 1$$

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