

16) $V: 2x + y - z = 0$, 上の $x = \begin{pmatrix} 6 \\ -1 \\ 3 \end{pmatrix}$ の正射影を求めよ.

答) $x = t, y = s$ とおくと $z = 2t + s$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ s \\ 2t+s \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix},$$

よって,

$$V = \text{span} \left(\underbrace{\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}_{\text{基底}} \right)$$

基底を ONB にして直交. $a_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, a_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ とおくと,

$$m_1 = \frac{1}{\|a_1\|} a_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}.$$

a_2 の m_1 方向の正射影を求めると,

$$\begin{aligned} (a_2)_p &= (a_2, m_1) m_1 = \left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right) \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \\ &= \frac{1}{5} (2) \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} \therefore (a_2)_L &= a_2 - (a_2)_p \\ &= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{5} \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}. \end{aligned}$$

$(a_2)_L$ を正規化して

$$m_2 = \frac{1}{\sqrt{4+25+1}} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{30}} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}.$$

$$\therefore V = \text{span} \left(\underbrace{\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{30}} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}}_{\text{ONB}} \right)$$

よって, x の V 上の正射影は

$$\begin{aligned} x_p &= (x, m_1) m_1 + (x, m_2) m_2 \\ &= \left(\begin{pmatrix} 6 \\ -1 \\ 3 \end{pmatrix}, \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right) \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \left(\begin{pmatrix} 6 \\ -1 \\ 3 \end{pmatrix}, \frac{1}{\sqrt{30}} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} \right) \frac{1}{\sqrt{30}} \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} \\ &= \frac{1}{5} \cdot 12 \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \frac{1}{30} (-12 - 5 + 3) \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} \\ &= \frac{1}{3} \begin{pmatrix} 10 \\ -7 \\ 13 \end{pmatrix}. \end{aligned}$$