線形数学Ⅱ

2008. 1. 28

問　$A = \begin{pmatrix} -7 & -6 \\ 18 & 14 \end{pmatrix}$ の A^k $(k=1,2,\cdots)$ を求めよ.

答　固有値は 2, 5.

$\lambda = 2$ の固有空間を求める.

$$\left(\begin{array}{cc|c} 2-(-7) & 6 & 0 \\ -18 & 2-14 & 0 \end{array}\right) \to \cdots \to \left(\begin{array}{cc|c} 1 & 2/3 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\therefore V(2) = V\left(\begin{pmatrix} -2/3 \\ 1 \end{pmatrix}\right)$$

$\lambda = 5$ の固有空間を求める.

$$\left(\begin{array}{cc|c} 5-(-7) & 6 & 0 \\ -18 & 5-14 & 0 \end{array}\right) \to \cdots \to \left(\begin{array}{cc|c} 1 & 1/2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\therefore V(5) = V\left(\begin{pmatrix} -1/2 \\ 1 \end{pmatrix}\right)$$

$P = \underline{\begin{pmatrix} -2/3 & -1/2 \\ 1 & 1 \end{pmatrix}}$ とおく $\longrightarrow$ $P^{-1} = \dfrac{1}{-\frac{2}{3}+\frac{1}{2}}\begin{pmatrix} 1 & 1/2 \\ -1 & -2/3 \end{pmatrix}$

$$= -6\begin{pmatrix} 1 & 1/2 \\ -1 & -2/3 \end{pmatrix} = \underline{\begin{pmatrix} -6 & -3 \\ 6 & 4 \end{pmatrix}}$$

以上から

$$A^k = P\begin{pmatrix} 2^k & 0 \\ 0 & 5^k \end{pmatrix}P^{-1}$$

$$= \begin{pmatrix} -2/3 & -1/2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2^k & 0 \\ 0 & 5^k \end{pmatrix}\begin{pmatrix} -6 & -3 \\ 6 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4\cdot 2^k - 3\cdot 5^k & 2\cdot 2^k - 2\cdot 5^k \\ -6\cdot 2^k + 6\cdot 5^k & -3\cdot 2^k + 4\cdot 5^k \end{pmatrix}$$