2007.5.16

(問) $\left(\dfrac{1}{g(x)}\right)'$ を求めよ.

(解) 定義にしたがって,

$$\frac{1}{h}\left(\frac{1}{g(x+h)}-\frac{1}{g(x)}\right)$$

$$=\frac{1}{h}\,\frac{g(x)-g(x+h)}{g(x+h)\,g(x)}$$

$$=-\left(\frac{g(x+h)-g(x)}{h}\right)\cdot\frac{1}{g(x+h)\,g(x)}.$$

$$\longrightarrow\ -g'(x)\cdot\frac{1}{g(x)^2}\quad(h\to 0)$$

$$\therefore\ \left(\frac{1}{g(x)}\right)'=-\frac{g'(x)}{g(x)^2}.$$

[問]　$(x^{-n})' = -n x^{n-1}$　$(n=1,2,3,\cdots)$

を商の微分公式から導け。

(答)　$(x^{-n})' = \left(\dfrac{1}{x^n}\right)' = \dfrac{(1)' x^n - 1\cdot(x^n)'}{x^{2n}}$

$= \dfrac{-n x^{n-1}}{x^{2n}} = -n x^{-n-1}$.

微積I

2007.5.16

(問) $$(\log x)' = \frac{1}{x} \quad (x > 0)$$

を定義から導け.

(答) $$\frac{\log(x+h) - \log x}{h}$$

$$= \frac{1}{h} \log\left(\frac{x+h}{x}\right) \quad (\because \text{対数法則})$$

$$\cancel{= \log\left(\frac{x+h}{x}\right)^{\frac{1}{h}} \quad (\because \text{―― 〃 ――})}$$

$$\cancel{= \left(1 + \frac{h}{x}\right)^{\frac{x}{h} \cdot \frac{1}{x}}}$$

$$= \frac{1}{x} \cdot \frac{x}{h} \log\left(\frac{x+h}{x}\right)$$

$$= \frac{1}{x} \log\left(1 + \frac{h}{x}\right)^{x/h} \quad (\because \text{対数法則})$$

$\frac{x}{h} = t$ とおくと $h \to +0$ のとき $t \to +\infty$,
$h \to -0$ のとき $t \to -\infty$

$$\therefore \lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{\frac{x}{h}} = \lim_{x \to \pm\infty} \left(1 + \frac{1}{t}\right)^t = e.$$

連続性から
$$\log\left(1 + \frac{h}{x}\right)^{\frac{x}{h}} \longrightarrow \log e = 1 \quad (h \to 0)$$

$$\therefore \lim_{h \to 0} \frac{\log(x+h) - \log x}{h} = \frac{1}{x}. \quad //$$

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(例) $\tan(\log(\cos(2x+1)))$ を微分せよ

(答)

$$\tan(\log(\cos(2x+1))) \quad \begin{cases} u = \tan w \\ w = \log z \\ z = \cos y \\ y = 2x+1 \end{cases}$$ の合成.

よって

$$\frac{du}{dx} = (\tan w)'(\log z)'(\cos y)'(2x+1)'$$

$$= \frac{1}{\cos^2 w} \cdot \frac{1}{z}(-\sin y)\cdot 2$$

$$= \frac{-2\sin(2x+1)}{\cos^2(\log(\cos(2x+1)))\cdot\cos(2x+1)}$$

//.