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[10] 不定積分を求めよ.

(1) $\int \frac{x^3+1}{x^2-4x+6}\,dx$ (2) $\int \frac{1}{(x^2+1)(x-1)^2}\,dx$

(答) (1) $\frac{x^3+1}{x^2-4x+6} = (x+4) + 5\cdot\frac{2x-4}{x^2-4x+6} - 3\cdot\frac{1}{(x-2)^2+\sqrt{2}^2}$

$\therefore \int \frac{x^3+1}{x^2-4x+6}\,dx = \frac{x^2}{2}+4x+5\log(x^2-4x+6) - \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right) + C.$

(2) $\frac{1}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-2)^2}$ と分解.

通分して,

$1 = (Ax+B)(x-1)^2 + C(x^2+1)(x-1) + D(x^2+1)$

連立方程式を立てて A, B, C, D を求めてもよいが, 以下のように求めてもよい。 ☞恒等式なので $x=1$ を代入すると,

$1 = D\cdot 2 \qquad \therefore D = \frac{1}{2}$

両辺を微分 $\Rightarrow 0 = A(x-1)^2 + (Ax+B)2(x-1) + 2Cx(x-1) + C(x^2+1) + x$

$x=1$ を代入 $\Rightarrow 0 = C\cdot 2 + 1 \qquad \therefore C = -\frac{1}{2}$

さらに両辺を微分 $\Rightarrow 0 = 2A(x-1) + A\cdot 2(x-1) + (Ax+B)\cdot 2 + 2C(x-1) + 2Cx + 2Cx + 1$

$x=1$ を代入 $\Rightarrow 0 = 2(A+B) + \underbrace{4C+1}_{-1} \qquad \therefore A+B = \frac{1}{2}.$

さらに微分 $\Rightarrow 0 = \underbrace{2A+2A+2A}_{6A} + \underbrace{2C+2C+2C}_{6C=-3} \qquad \therefore A = \frac{1}{2}$

$\therefore B = 0$

与式 $= \int \frac{\frac{1}{2}x}{x^2+1}\,dx + \int \frac{-\frac{1}{2}}{x-1}\,dx + \int \frac{\frac{1}{2}}{(x-1)^2}\,dx$

$= \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log|x-1| - \frac{1}{2}\cdot\frac{1}{(x-1)} + C.$