微分積分学 I

2007 年 8 月 1 日

[**問**]: 次の定積分を計算せよ.

(1) $\int_1^2 (2x^3 - \dfrac{3}{\sqrt[4]{x}} + x^{5/2})dx$

(2) $\int_0^{\pi/2} x^2 \cos x \, dx$

(3) $\int_{-2}^1 \frac{1}{x^2+x+1} dx$

(4) $\int_0^1 \frac{x}{\sqrt{2-x^2}} dx$

(5) $\int_{-3}^2 (x+3)^4 (x-2)^3 dx$ を置換積分，部分積分の２通りの方法で計算せよ.

[解答]: (1) $= [\frac{x^4}{2} - 3\frac{4}{3}x^{3/4} + \frac{2}{7}x^{7/2}]_1^2 = \frac{157}{14} - 2^{11/4} + \frac{2^{9/2}}{7}$

(2) $= [x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\pi/2} = \frac{\pi^2}{4} - 2$

(3) $= [\frac{2}{\sqrt{3}} \tan^{-1}(\frac{2x+1}{\sqrt{3}})]_{-2}^1 = \frac{4}{3\sqrt{3}}\pi$

(4) $= [-\sqrt{2-x^2}]_0^1 = \sqrt{2} - 1$

(5) 部分積分による方法：

$$\begin{aligned}
\int_{-3}^2 (x+3)^4 (x-2)^3 dx &= \frac{(x+3)^5}{5}(x-2)^3 - \frac{1}{5}\int (x+3)^5 3(x-2)^2 dx \\
&= -\frac{3}{5}\int (x+3)^5 (x-2)^2 dx \\
&= \cdots \\
&= -\frac{1}{280}[(x+3)^8]_{-3}^2 = -\frac{5^8}{280}
\end{aligned}$$

置換積分のよる方法：$x+3=t$ とおく.

$$\begin{aligned}
\int (x+3)^4 (x-2)^3 dx &= \int (x+3)^4 (x+3-5)^3 dx \\
&= \int t^4 (t-5)^3 dt \\
&= \frac{t^8}{8} - \frac{15}{7}t^7 + \frac{75}{6}t^6 - \frac{125}{5}t^5 \\
&= \frac{(x+3)^8}{8} - \frac{15}{7}(x+3)^7 + \frac{75}{6}(x+3)^6 - \frac{125}{5}(x+3)^5
\end{aligned}$$

よって

$$\begin{aligned}
\int (x+3)^4 (x-2)^3 dx &= [\frac{(x+3)^8}{8} - \frac{15}{7}(x+3)^7 + \frac{75}{6}(x+3)^6 - \frac{125}{5}(x+3)^5]_{-3}^2 \\
&= \frac{-5^8}{280}
\end{aligned}$$

K.U.