微分積分学II

2007.11.1

問　$z = f(x, y)$ ，$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ で合成関数を考える．

z_θ ，z_{rr} ，$z_{r\theta}$ ，$z_{\theta\theta}$ を求めよ．

答　$z_r = f_x \cdot \cos\theta + f_y \cdot \sin\theta$

$$z_{rr} = \left\{ (f_x)_x \cos\theta + (f_x)_y \sin\theta \right\} \cdot \cos\theta + \left\{ (f_y)_x \cos\theta + (f_y)_y \sin\theta \right\} \cdot \sin\theta$$

$$= f_{xx} \cos^2\theta + 2 f_{xy} \sin\theta\cos\theta + f_{yy} \cdot \sin^2\theta$$

$$z_{r\theta} = (z_r)_\theta$$

$$= \left\{ (f_x)_x (-r\sin\theta) + (f_x)_y \, r\cos\theta \right\} \cos\theta + f_x \cdot (-\sin\theta)$$

$$+ \left\{ (f_y)_x (-r\sin\theta) + (f_y)_y \, r\cos\theta \right\} \sin\theta + f_y \cdot \cos\theta$$

$$= -f_{xx} \, r\sin\theta\cos\theta + f_{xy} \cdot r \cdot (\cos^2\theta - \sin^2\theta) + f_{yy} \, r\sin\theta\cos\theta$$

$$- f_x \sin\theta + f_y \cdot \cos\theta$$

$$z_\theta = f_x \cdot (-r\sin\theta) + f_y \, r\cos\theta$$

$$z_{\theta\theta} = \left\{ (f_x)_x (-r\sin\theta) + (f_x)_y \, r\cos\theta \right\} (-r\sin\theta) + f_x (-r\cos\theta)$$

$$+ \left\{ (f_y)_x (-r\sin\theta) + (f_y)_y \, r\cos\theta \right\} (\overset{r\cos\theta}{\cancel{r\sin\theta}}) + f_y (-r\sin\theta)$$

$$= f_{xx} r^2 \sin^2\theta - 2 f_{xy} r^2 \sin\theta \cdot \cos\theta + f_{yy} r^2 \cos^2\theta$$

$$- f_x r\cos\theta - f_y r\sin\theta .$$