問：(1) $\iint_D e^{-x}\cos y\,dxdy$, $D=\{1\leqq x\leqq 2,\ 0\leqq y\leqq \frac{\pi}{2}\}$

(2) $\iint_D \frac{y}{x}\,dxdy$, $D=\{1\leqq x\leqq 2,\ 0\leqq y\leqq 1\}$

(3) $\iint_D e^{2x+y}\,dxdy$, $D=\{0\leqq x\leqq 1,\ 1\leqq y\leqq 2\}$

(4) $\iint_D \sin(x-y)\,dxdy$, $D=\{0\leqq x\leqq \frac{\pi}{2},\ 0\leqq y\leqq \frac{\pi}{2}\}$

答：(1)

$$\iint_D e^{-x}\cos y\,dxdy=\int_0^{\pi/2}\left(\int_1^2 e^{-x}\cos y\,dx\right)dy$$

$$=\int_0^{\frac{\pi}{2}}\left[-e^{-x}\cos y\right]_1^2 dy$$

$$=\int_0^{\frac{\pi}{2}}(-e^{-2}\cos y+e^{-1}\cos y)\,dy$$

$$=(e^{-1}-e^{-2})\left[\sin y\right]_0^{\pi/2}=\underline{e^{-1}-e^{-2}}.$$

(2)

$$\iint_D \frac{y}{x}\,dxdy=\int_1^2\left(\int_0^1 \frac{y}{x}\,dy\right)dx$$

$$=\int_1^2\left[\frac{y^2}{2x}\right]_0^1 dx$$

$$=\int_1^2 \frac{1}{2x}\,dx$$

$$=\left[\frac{1}{2}\log|x|\right]_1^2=\underline{\frac{1}{2}\log 2}.$$

$x\to y$の順に行うと，

$$\iint_D \frac{y}{x}\,dxdy=\int_0^1\left(\int_1^2 \frac{y}{x}\,dx\right)dy$$

$$=\int_0^1\left[y\log|x|\right]_1^2 dy$$

$$=\int_0^1 y\log 2\,dy$$

$$=(\log 2)\cdot\left[\frac{y^2}{2}\right]_0^1=\underline{\frac{1}{2}\log 2}.$$

(3)

$$\iint_D e^{2x+y}\,dx\,dy = \int_0^1 \left(\int_1^2 e^{2x+y}\,dy \right) dx$$

$$= \int_0^1 \left[e^{2x+y} \right]_1^2 dx$$

$$= \int_0^1 \left(e^{2x+2} - e^{2x+1} \right) dx$$

$$= \left[\frac{1}{2} e^{2x+2} - \frac{1}{2} e^{2x+1} \right]_0^1$$

$$= \frac{1}{2}\left\{ e^4 - e^3 - e^2 + e \right\}.$$

(4)

$$\iint_D \sin(x-y)\,dx\,dy = \int_0^{\frac{\pi}{2}} \left(\int_0^{\frac{\pi}{2}} \sin(x-y)\,dx \right) dy$$

$$= \int_0^{\frac{\pi}{2}} \left[-\cos(x-y) \right]_0^{\pi/2} dy$$

$$= \int_0^{\frac{\pi}{2}} \left(-\cos\left(\frac{\pi}{2}-y\right) + \underbrace{\cos(-y)}_{\cos y} \right) dy$$

$$= \left[+\sin\left(\frac{\pi}{2}-y\right) + \sin y \right]_0^{\frac{\pi}{2}}$$

$$= 1 - 1 = 0.$$