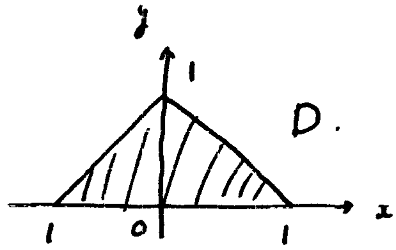
問 $\iint_D xy^2\,dx\,dy$



$x \to y$ の順に累次積分せよ.

答

$$\int_0^1\left(\int_{y-1}^{-y+1} xy^2\,dx\right)dy$$

$$=\int_0^1 \left[\frac{x^2y^2}{2}\right]_{y-1}^{-y+1}dy$$

$$=\int_0^1\left\{\frac{(-y+1)^2y^2}{2}-\frac{(y-1)^2y^2}{2}\right\}dy=0.$$

問 累次積分の順序を交換せよ.

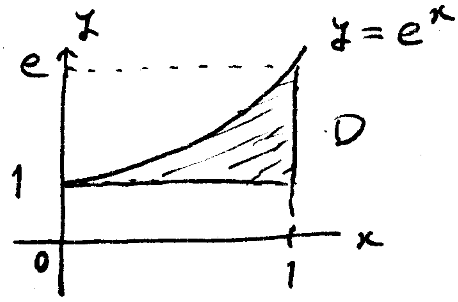
(1) $\int_0^1\left(\int_1^{e^x} f(x,y)\,dy\right)dx$

(2) $\int_{-1}^1\left(\int_0^{\sqrt{1-x^2}} f(x,y)\,dy\right)dx$

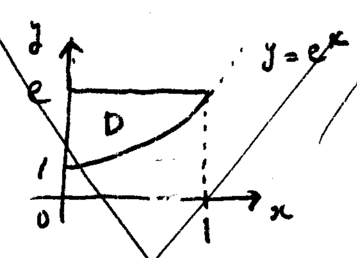
(3) $\int_{-1}^1\left(\int_{y-1}^{y+1} f(x,y)\,dx\right)dy$

(4) $\int_0^4\left(\int_{\sqrt{y}}^2 f(x,y)\,dx\right)dy$

(5) $\int_2^3\left(\int_1^{x^2} f(x,y)\,dy\right)dx$



答: (1)

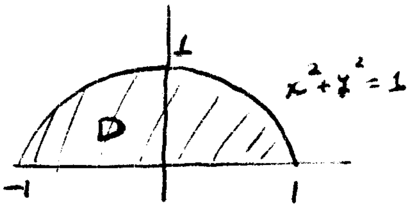


$$=\int_1^e\left(\int_0^{\log y} f(x,y)\,dx\right)dy$$

$$=\int_1^e\left(\int_{\log y}^1 f(x,y)\,dx\right)dy$$

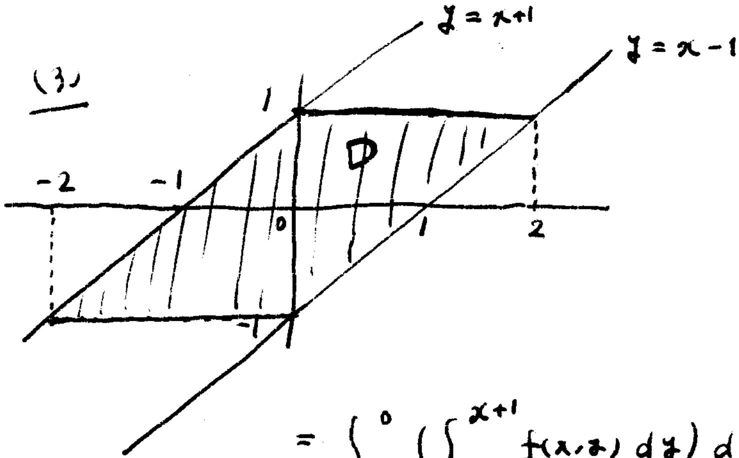
$$\cancel{\int_0^1\left(\int_0^1 f(x,y)\,dx\right)dy}$$

(2)



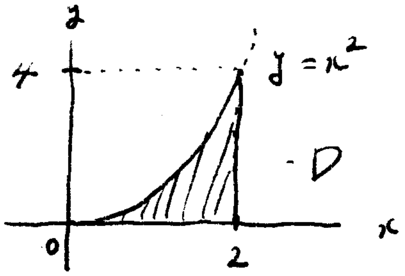
$$= \int_0^1 \left(\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx \right) dy$$

(3)



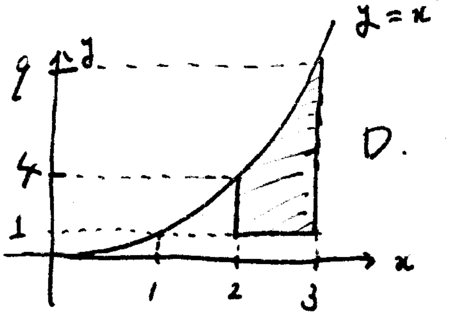
$$= \int_{-2}^{0} \left(\int_{-1}^{x+1} f(x,y)\,dy \right) dx + \int_0^2 \left(\int_{x-1}^{1} f(x,y)\,dy \right) dx$$

(4)



$$= \int_0^2 \left(\int_0^{x^2} f(x,y)\,dy \right) dx$$

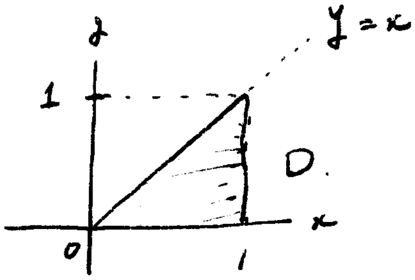
(5)



$$= \int_1^4 \left(\int_2^3 f(x,y)\,dx \right) dy + \int_4^9 \left(\int_{\cancel{\sqrt{x}}\ \sqrt{y}}^{3} f(x,y)\,dx \right) dy$$

例　積分の順序交換をして値を求めよ．

$$\int_0^1 \left(\int_y^1 x^2 y \, dx \right) dy$$



$$= \int_0^1 \left(\int_0^x x^2 y \, dy \right) dx$$

$$= \int_0^1 \left[\frac{x^2 y^2}{2} \right]_0^x dx$$

$$= \int_0^1 \frac{x^4}{2} dx = \left[\frac{1}{10} x^5 \right]_0^1 = \frac{1}{10}.$$