

1. $z = \log(3x + y^3)$, $x = -3uv$, $y = 2u^2 + v^3$ のとき, $\frac{\partial z}{\partial v}$ を求めよ.

[解答]:

$$\begin{aligned}\frac{\partial z}{\partial v} &= z_x x_v + z_y y_v \\ &= \frac{3}{3x + y^3}(-3u) + \frac{3y^2}{3x + y^3}(3v^2)\end{aligned}$$

あとは u, v の式で整理する.

2. $z = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$ のとき, $z_{\theta r}$ を求めよ.

[解答]:

$$z_{\theta} = f_x x_{\theta} + f_y y_{\theta} = -r \sin \theta f_x + r \cos \theta f_y.$$

積の微分公式から

$$\begin{aligned}z_{\theta r} &= (z_{\theta})_r \\ &= -\sin \theta f_x + (-r \sin \theta)(f_{xx} x_r + f_{xy} y_r) + \cos \theta f_y + r \cos \theta (f_{yx} x_r + f_{yy} y_r) \\ &= -r \sin \theta \cos \theta f_{xx} - r \sin^2 \theta f_{xy} + r \cos^2 \theta f_{yx} + r \sin \theta \cos \theta f_{yy} - \sin \theta f_x + \cos \theta f_y.\end{aligned}$$

K. U.