6/21 微積Ⅱ

問: $z = f(x, y)$, $\begin{cases} x = a + th \\ y = b + tk \end{cases}$ のとき $\dfrac{dz}{dt}$, $\dfrac{d^2z}{dt^2}$ を求めよ.

$$\frac{dz}{dt} = f_x \cdot x_t + f_y \cdot y_t = f_x(a+th, b+tk) \times h + f_y(a+th, b+tk) \times k$$

$$\frac{d^2z}{dt^2} = (f_{xx} \cdot x_t + f_{xy} \cdot y_t)h + (f_{yx} \cdot x_t + f_{yy} \cdot y_t)k$$

$$= (f_{xx} \cdot h + f_{xy} k)h + (f_{yx} h + f_{yy} k)k$$

$$= h^2 f_{xx}(a+th, b+tk) + hk\, f_{xy}(a+th, b+tk)$$

$$+ hk\, f_{yx}(a+th, b+tk) + k^2 f_{yy}(a+th, b+tk)$$

問: $z = e^{-x}\cos y$ の $a = b = 0$ における 2次のテイラーの定理をかけ.

$$f(h,k) = f(0,0) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(0,0) + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(\theta h, \theta k),$$

$$0 < \theta < 1.$$

$f(0,0) = 1$,

$f_x = -e^{-x}\cos y \quad \therefore f_x(0,0) = -1$

$f_y = -e^{-x}\sin y \quad \therefore f_y(0,0) = 0$

$f_{xy} = e^{-x}\sin y \quad \therefore f_{xy}(\theta h, \theta k) = e^{-\theta h}\sin\theta k$

$\qquad\qquad\qquad\qquad\quad (= f_{yx}(\theta h, \theta k))$

$f_{xx} = e^{-x}\cos y \quad \therefore f_{xx}(\theta h, \theta k) = e^{-\theta h}\cos\theta k$

$f_{yy} = -e^{-x}\cos y \quad \therefore f_{yy}(\theta h, \theta k) = -e^{-\theta h}\cos\theta k$

よって

$$f(h,k) = 1 - h + \frac{1}{2}\left(h^2 e^{-\theta h}\cos\theta k + 2hk\, e^{-\theta h}\sin\theta k - k^2 e^{-\theta h}\cos\theta k\right), \quad 0 < \theta < 1.$$