6/28 微積 II.

例 $z = \log(1+2x-y)$ の 2次のマクローリンの定理をかけ

$$z_x = \frac{2}{1+2x-y}, \quad z_y = \frac{-1}{1+2x-y}$$

$$z_{xx} = \frac{-4}{(1+2x-y)^2} \qquad z_{xy} = \frac{2}{(1+2x-y)^2}$$

$$z_{yy} = \frac{-1}{(1+2x-y)^2}$$

よって,

$$z = z(0) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) z(0.0) + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 z(\theta h, \theta k)$$

$$(0<\theta<1)$$

$$= 2h - k + \frac{1}{2}\left(h^2 \frac{-4}{(1+2\theta h - \theta k)^2} + 2hk \frac{2}{(1+2\theta h - \theta k)^2} \right.$$

$$\left. + k^2 \frac{-1}{(1+2\theta h - \theta k)^2} \right), \quad 0<\theta<1$$

$$= 2h - k - \frac{2h^2}{(1+2\theta h - \theta h)^2} + \frac{2hk}{(1+2\theta h - \theta k)^2} - \frac{1}{2}\cdot\frac{k^2}{(1+2\theta h - \theta k)^2}$$