7/5 微積II

問：$f(x,y) = x^2 - 2xy + 6y^2 - 3x + y$ の停留点をもとめよ

$$f_x = 2x - 2y - 3 = 0$$
$$+\big)\ f_y = -2x + 12y + 1 = 0$$
$$10y - 2 = 0 \quad \therefore y = \frac{1}{5}$$

1式へ代入

$$2x = 2y + 3 = \frac{2}{5} + 3 = \frac{17}{5}$$
$$\therefore x = \frac{17}{10}$$
$$(x, y) = \left(\frac{17}{10}, \frac{1}{5}\right).$$

問：$f(x,y) = x^2 - 2xy + 4y^2 - 2x + y$ の極値をもとめよ.

まず停留点をもとめる.

$$f_x = 2x - 2y - 2 = 0$$
$$+\big)\ f_y = -2x + 8y + 1 = 0$$
$$6y - 1 = 0 \quad \therefore y = \frac{1}{6}$$

1式へ代入,

$$2x = 2y + 2 = \frac{1}{3} + 2 = \frac{7}{3}$$
$$\therefore x = \frac{7}{6}$$
$$(x, y) = \left(\frac{7}{6}, \frac{1}{6}\right).$$

得られた停留点が極値点か否かを調べる.

$$f_{xx} = 2, \quad f_{yy} = 8 \quad f_{xy} = -2$$

$$\therefore \Delta = f_{xy}^2 - f_{xx} f_{yy} = 4 - 2 \cdot 8 = -12 < 0.$$

さらに $f_{xx}\left(\frac{7}{6}, \frac{1}{6}\right) = 2 > 0$ であるから

$f\left(\frac{7}{6}, \frac{1}{6}\right)$ は極小値である

$$= -\frac{13}{12}.$$