

14) $y' + \frac{x^2}{x^2-1} = 0$ の一般解?

$$\int \frac{1}{y} dy = \int -\frac{x}{x^2-1} dx + C$$

$$\log|y| = -\frac{1}{2} \log|x^2-1| + C$$

$$\therefore \log|y^2(x^2-1)| = C$$

$$\therefore y^2(x^2-1) = C \quad \therefore y^2 = \frac{C}{x^2-1}$$

$$\int \frac{f(x)}{f(x)} dx = \log|f(x)| + C$$

15) $y' + y = e^{2x}$

2次" $y' + y = 0$ の解 $y = ce^{-x}$

$C=1$ の解 $y_0 = e^{-x}$

y の一般解 $u = \frac{y}{y_0} = ce^{3x}$

2次" $y' = (uy_0)' = u'y_0 + uy_0' = u'y_0 + u_0(-y_0)$
 $= u'y_0 - y'$

$$\therefore y' + y = u'y_0$$

$$\therefore e^{2x} = u'y_0 \quad \therefore u' = \frac{e^{2x}}{y_0} = \frac{e^{2x}}{e^{-x}} = e^{3x}$$

2次" $u = \int e^{3x} dx + C = \frac{1}{3} e^{3x} + C$

2次" $y =$

$$y = uy_0$$

$$= \left(\frac{1}{3} e^{3x} + C\right) e^{-x} = \frac{1}{3} e^{2x} + ce^{-x}$$